From the solutions for short and long time, it may be apparent that the enhancement of evaporation is proportional to $\sqrt{Pe}$, which, in turn, is proportional to the electric field strength. Although the results are encouraging, the current treatment is only applicable to the situations where the electric field generates a Taylor type of motion. At high field strength the strong current would create a magnetic field and a nonvanishing body force in the momentum equation. Such couplings and interactions must be accounted for in the analysis.

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Radiation View Factors from a Sphere to Nonintersecting Planar Surfaces

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Introduction

IN the calculation of radiative heat exchange between two bodies, the determination of a radiation view factor becomes essential. Although a large number of radiation shape factors for different configurations have been made available, due to the tedious task of quadruple integral, there still exist many important geometric configurations involving a sphere and a finite surface for which the associated view factors are not known.

The purpose of this study is to develop the radiation shape factors systematically from a sphere to a class of nonintersecting planes. Radiation between a sphere and a planar element was evaluated analytically by Chung and Sumitra[1] and Juul.[2] Feingold and Gupta[3] developed an analytical approach for the radiation view factors from spheres to various nonintersecting coaxial planes. Complementary to their work, Chung and Naraghi[4] developed closed-form solutions for the view factors from spheres to intersecting disks and to the external surface of a coaxial cylinder. They further developed a general formulation for the view factors between a sphere and a class of axisymmetrical bodies.[5] Balance and Donovan[6] studied the view factor between spheres and intersecting disks using a Monte Carlo method.

A logical continuation of the previous efforts would be the evaluation of the view factors from a sphere to any noncoaxial surfaces. To the best knowledge of these authors, the aforementioned case has not been solved.

Mathematical Formulation

Consideration is given to Fig. 1, which depicts the radiation from a sphere to a noncoaxial differential planar area; its plane does not intersect the sphere. The view factor from dA_1 to A_2 has been found[4] as

$$F_{dA_1-A_2} = \frac{\cos\theta}{(1+S)^2}, \quad \theta \le \cot^{-1}\left(\frac{r_s}{\sqrt{(s+r_s)^2 - r_s^2}}\right) \tag{1}$$

where

$$S = \frac{s}{r_s}$$

Referring to Fig. 1, we have $\cos\theta = \mathrm{d}/\sqrt{x^2+y^2+\mathrm{d}^2}$ and $(1+S)^2 = (x^2+y^2+\mathrm{d}^2)/r_s^2$. Substituting the expressions into Eq. (1), making use of the reciprocity rule, and carrying out the integration gives

$$F_{2-1} = \iint_{A_1} \mathrm{d}F_{A_2-\mathrm{d}A_1} = \frac{\mathrm{d}}{4\pi}\iint_{A_1} \frac{\mathrm{d}A_1}{(x^2+y^2+\mathrm{d}^2)^{3/2}} \tag{2}$$

If A_1 is symmetrical with respect to the x or y axis, Eq. (2) can be reduced to a single integration form. Suppose that A_1 is

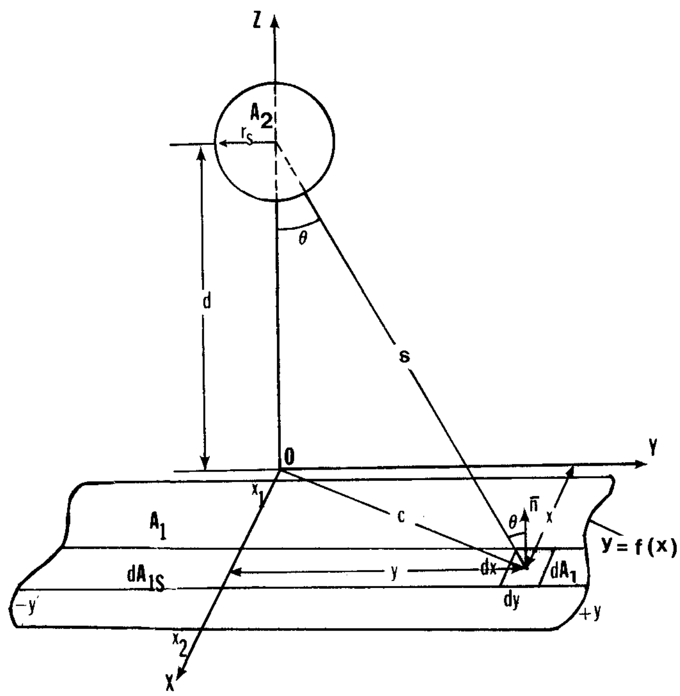

Fig. 1 Radiation between a sphere and a planar surface.

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symmetrical with respect to the x axis. Integrating Eq. (2) with respect to y, we obtain the view factor from a sphere to an infinite strip, with a width equal to dx. Knowing that $y = f(x)$ and integrating the resulting expression from x_1 to x_2, we obtain the view factor from a sphere to any nonintersecting planar surfaces:

$$F_{2-1} = \frac{\mathrm{d}}{2\pi}\int_{x_1}^{x_2} \frac{f(x)}{(x^2+\mathrm{d}^2)\sqrt{x^2+[f(x)]^2+\mathrm{d}^2}}\,\mathrm{d}x \tag{3}$$

Specific cases in light of the general expression presented are compiled in Table 1 for different $f(x)$.

Results and Discussion

It can be seen that the expressions represented in Table 1 require only a single integration, which can be performed easily using the standard numerical techniques. The results for all of the limiting cases such as the view factors from a sphere to a coaxial disk, sector, segment, rectangle, and a noncoaxial disk, can be obtained directly from Table 1. As an example, if we let $\alpha = 2\pi$ in case 1, we obtain the view factor from a sphere to a noncoaxial disk in the form of

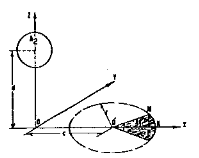
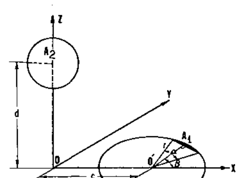
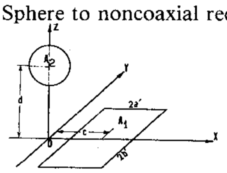
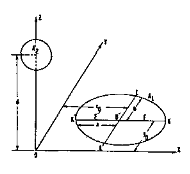
$$F_{2-1} = \frac{R^2}{2\pi}\int_{Z-1}^{Z+1} \frac{\sqrt{1-(X-Z)^2}}{(1+R^2X^2)\sqrt{1+R^2X^2+R^2-R^2(X-Z)^2}}\,\mathrm{d}X \tag{4}$$

The numerical results of this expression for various values of Z agree excellently with those of Ref. 3. If we set $Z = c/r = 0$ in the preceding equation, it yields the view factor from sphere to a coaxial disk in the form of[5]

$$F_{2-1} = \frac{1}{2}\left[1 - \frac{1}{(1+R^2)^{1/2}}\right] \tag{5}$$

which is identical to the expression derived by Feingold and Gupta.[3] Expressions listed in Table 1 have been solved numerically using Gaussian quadrature method for some arbitrarily chosen parameters. Typical results are illustrated in Figs. 2–5.

Table 1 View factors from sphere to some nonintersecting planes

Configuration ID	View factor from the sphere, F_{2-1}	Parameters
1) Sphere to noncoaxial sector 	$m\left[\int_{X_2}^{X_1} K_1\mathrm{d}X + \int_{X_1}^{X_3} K_2\mathrm{d}X\right]$	$K_i = \frac{f_i}{h\sqrt{h+R^2f_i^2}}$ $i = 1,2,3,4$ $h = 1 + R^2X^2$ $f_1 = (X-Z)\tan\frac{\alpha}{2}$ $f_2 = \sqrt{1-(X-Z)^2}$ $f_3 = \frac{\cos\left(\frac{\alpha}{2}-\beta\right)+Z-X}{\tan\beta} - \sin\left(\frac{\alpha}{2}-\beta\right)$ $f_4 = \frac{\cos\left(\frac{\alpha}{2}+\beta\right)+Z-X}{\tan\beta} + \sin\left(\frac{\alpha}{2}+\beta\right)$ $X_1 = Z + \cos\frac{\alpha}{2}$, $X_2 = Z$ $X_3 = Z + 1$, $X_4 = Z + \cos\left(\frac{\alpha}{2}-\beta\right)$ $X_5 = Z + \cos\left(\frac{\alpha}{2}+\beta\right)$, $X_6 = Z - 1$ $Z = \frac{c}{r}$, $R = \frac{r}{\mathrm{d}}$ $m = \frac{R^2}{2\pi}$
2) Sphere to noncoaxial segment 	$\frac{m}{2}\left[\int_{X_5}^{X_3} K_2\mathrm{d}X - \int_{X_5}^{X_4} K_3\mathrm{d}X + \int_{X_4}^{X_3} K_2\mathrm{d}X\right]$ for $\beta \le \frac{\alpha}{2} \le \frac{\pi}{2}$ $\frac{m}{2}\left[\int_{X_5}^{X_4} (K_2 - K_3)\mathrm{d}X\right]$ for $\alpha \le \pi$ and $\beta \ge \frac{\alpha}{2}$ or $(\pi-\beta) \ge \frac{\alpha}{2}$ $\frac{m}{2}\left[\int_{X_6}^{X_4} K_2\mathrm{d}X - \int_{X_5}^{X_4} K_4\mathrm{d}X + \int_{X_6}^{X_5} K_2\mathrm{d}X\right]$ for $(\pi-\beta) \le \frac{\alpha}{2} \le \frac{\pi}{2}$	
3) Sphere to noncoaxial rectangle 	$\frac{B_1}{2\pi}\int_{Z-B_2}^{Z+B_2} \frac{\mathrm{d}X}{(1+X^2)\sqrt{1+X^2+B_1^2}}$	$B_1 = \frac{b'}{\mathrm{d}}$, $B_2 = \frac{a'}{d}$, $Z = \frac{c}{\mathrm{d}}$
4) Sphere to noncoaxial ellipse 	$\frac{E^2}{4\pi}\int_{X_o-1}^{X_o+1} \frac{1}{p}(q_1 - q_2)\mathrm{d}X$	$q_i = u_i/\sqrt{p+E^2u_i^2}$, $i = 1,2$ $p = 1 + E^2X^2$ $u_1 = y_o + \sqrt{(1-\epsilon^2)[1-(X-X_o)^2]}$ $u_2 = y_o - \sqrt{(1-\epsilon^2)[1-(X-X_o)^2]}$ $E = a/\mathrm{d}$, $\epsilon = \sqrt{1-b^2/a^2}$ $X_o = x_o/a$, $Y_o = y_o/a$

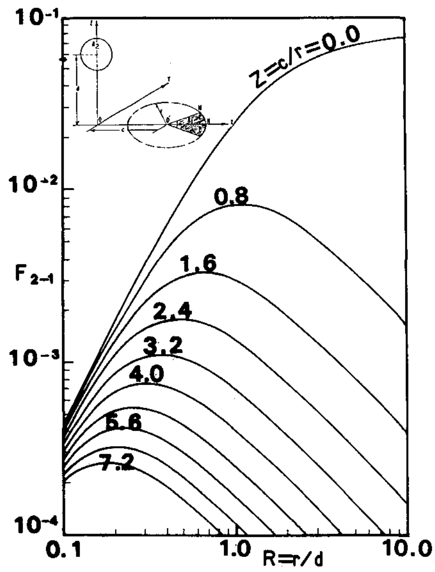


Fig. 2 View factor from a sphere to a sector, $\alpha = 60$ deg.

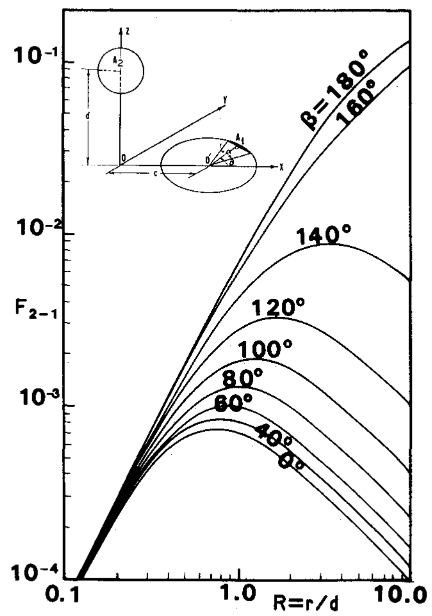


Fig. 3 View factor from a sphere to a segment, $\alpha = 60$ deg $Z = 1$.

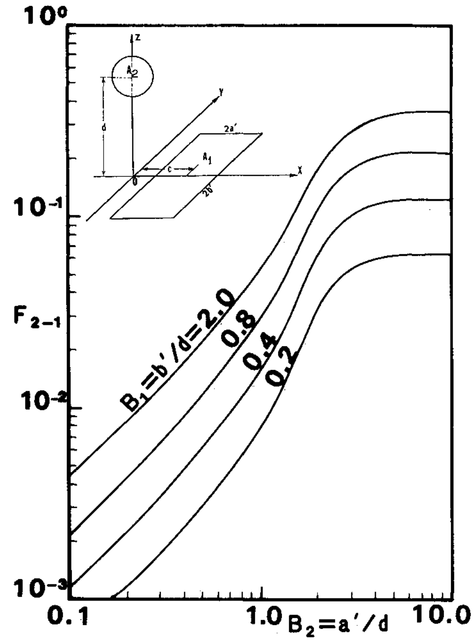


Fig. 4 View factor from a sphere to a rectangle with $Z = 2$.

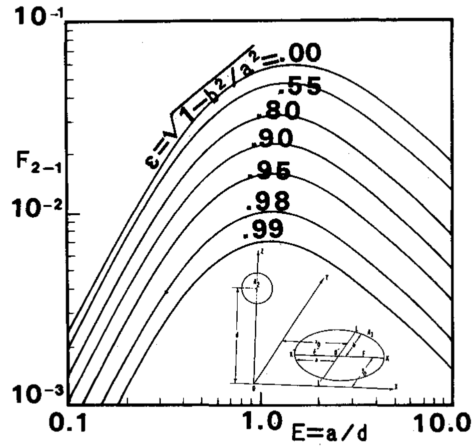


Fig. 5 View factor from a sphere to a noncoaxial ellipse, $x_o = y_o = 1$.

Figure 2 plots the view factor from a sphere to a sector against $R(R = r/d)$ with $Z(Z = c/r)$ as a parameter and the sector angle α fixed at 60 deg. As can be seen, for $Z = 0$ (coaxial sector) and at a very large value of R, the curve becomes asymptote to $F_{2-1} = 0.5 \times 60/360 = 0.083$. Figure 3 shows the view factor from a sphere to a nonsymmetric segment as a function of R with $\alpha = 60$ deg. For a constant Z, as β increases, i.e., the segment moves closer to the sphere, the view factor increases. Figure 4 illustrates the view factor from a sphere to a rectangle in terms of $B_2(B_2 = a'/d)$ and $B_1(B_1 = b'/d)$ for $Z = 2$. The curves in this situation eventually become asymptote to some constant values. Figure 5 gives the view factor from a sphere to an ellipse. For a coaxial ellipse, i.e., $x_o = y_o = 0$, as E becomes very large, all curves approach a constant value of 0.5. This is in agreement with our intuition. Additional numerical results may be found in Ref. 7 (through interlibrary loan).

The application of the present general formula is not restricted to the geometries mentioned. It can also be applied to generate the radiation view factors from a sphere to any other nonintersecting planar surfaces, such as a triangle, parallelogram, and trapezoid, either in a coaxial or noncoaxial situation.

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